TR/PA/92/86

Users' Guide for the Harwell-Boeing Sparse Matrix Collection (Release I)

Iain S. Duff
CERFACS, 42 Ave G. Coriolis, 31057 Toulouse Cedex, France *

Roger G. Grimes and John G. Lewis
Research and Technology Division, Boeing Computer Services,
Mail Stop 7L-21, P. O. Box 24236, Seattle, WA 98124-0346, USA. †

October 1992

Abstract

We describe the complete set of matrices in the Harwell-Boeing sparse matrix collection, a set of standard test matrices for sparse matrix problems. This description includes some documentation for each matrix (or set of matrices) in the collection. We also describe how a copy of the collection may be obtained.

Keywords

sparse matrices, test matrices, matrix collection

1 Introduction

This users' guide is a description of the matrices currently in the Harwell-Boeing Sparse Matrix Collection. It is our intention to include further problems and additional matrix generating subprograms in the future. We will produce subsequent releases of this report from time to time as the collection evolves.

This users' guide describes the distribution process for the collection. An overview of the collection is given in [1] which also provides some historical comments and guidelines for

*also at Rutherford Appleton Laboratory, Oxon OX11 0QX, England

†Supported in part by AFOSR grant F49620-87-C-0037

contributions to the collection. Here we describe the mechanism for getting a copy of the matrices and for using the collection. The detailed contents of the collection are described in two appendices. Appendix A describes the representation used to store the matrices. Appendix B is a collection of descriptions for each set of matrices in the collection.

2 How to obtain the collection

The collection of sparse matrices can be obtained in two ways. The collection is available through direct electronic file transfer, by anonymous ftp to CERFACS. Alternatively the complete collection can be obtained on Unix tar tapes from Roger Grimes at Boeing.

The most flexible method is electronic, using anonymous ftp. The requester should use ftp to connect to orion.cerfacs.fr (130.63.200.33). Log on using the identifier "*anonymous*" and use as password your email address. The collection is in the directory pub/harwell_boeing. The matrices are grouped together according to the sets in Table 1, which correspond to the documentation in Appendix B. The matrices are in compressed form, so a binary command should be given to ftp before the files are fetched. Each set of matrices is designated by xxxx.data.Z where xxxx is the name of the set (in lower case) as given in Table 2.

A sample ftp session is given below, with the required responses in **boldface**:

% **ftp orion.cerfacs.fr**
Connected to orion.cerfacs.fr.
220 orion FTP server (SunOS 4.1) ready.
Name (orion.cerfacs.fr:jglewis): **anonymous**
331 Guest login ok, send ident as password.
Password: **jglewis@...**
230 Guest login ok, access restrictions apply.
ftp> **cd pub/harwell_boeing**
250 CWD command successful.
ftp> **binary**
200 Type set to I.
ftp> **ls**
200 PORT command successful.
150 ASCII data connection for /bin/ls (130.42.28.80,2469) (0 bytes).
acoust.data.Z
airtfc.data.Z
astroph.data.Z
bcspwr.data.Z
⋮
ftp> **mget coun*.Z**
mget counterx.data.Z? **y**
200 PORT command successful.
150 Binary data connection for counterx.data.Z (130. ...
226 Binary Transfer complete.
local: counterx.data.Z remote: counterx.data.Z

701 bytes received in 1.7 seconds (0.41 Kbytes/s)
ftp> **quit**

Distribution on tape is available for those who cannot use ftp. This service is available by sending a high density (150 Megabyte) 1/4" cartridge tape (e.g. 3M DC 6150) to Roger Grimes. The tape will be written as a Unix tar tape, with the matrices as individual compressed files in a single directory.

Table 1: Sets of Matrices

acoust	bcsstruc5	counterx	jagmesh	lsq	psadmit
airtfc	bcsstruc6	dwt	lanpro	manteuffel	psmigr
astroph	cannes	econaus	laplace	nnceng	saylor
bcspwr	cegb	econiea	lapu	nucl	sherman
bcsstruc1	chemimp	facsimile	lns	oilgen	smtape
bcsstruc2	chemwest	gemat	lockheed	platz	steam
bcsstruc3	cirphys	grenoble	lshape	pores	watt
bcsstruc4					

3 Matrix format

We have chosen a particular format in which to represent the sparse matrices in the collection. The major advantage of our format is that it is general and provides a standard, although it is sometimes not the best representation for using available sparse matrix codes or for research purposes. The format in which each matrix is held is given in Appendix A. In this appendix we give program fragments that indicate how the user can read the matrices.

4 Possible future enhancements

In our pre-release that accompanied the paper [1], Roger Grimes offered the possibility of obtaining subsets of the collection through various sources, categories, or keywords. These are shown in Tables 2 to 4. However, there was so little demand for this service, that he now does not offer it. Also the advent of anonymous ftp gives more control to the user of the collection. However, we would like to hear of ways in which you feel access to the collection might be improved.

Table 2: Sources for Matrices

Abbas - Newcastle	1	Manteuffel - Sandia	1
Appleyard - Harwell	3	Marro - Cannes	18
Ashkenazi-Nottingham	6	Natl. Nuclear Corp.	4
BCS	68	Pearson - Australia	2
Burchett - GE	3	Platzman - Chicago	4
Cachard - Grenoble	7	Saunders - SOL	20
Carlsson - Oslo	2	Saylor - Illinois	3
Curtis - Harwell	16	Sherman - Nolan	5
Donovan - CEGB	4	Simon - BCS	7
Erisman - BCS	1	Slater - UCSB	3
Everstine - DWTNSRD	30	Somerville - Watt U.	2
Gentleman - Waterloo	1	Szyld - NYU	9
George - Waterloo	21	Tylavsky - ASU	4
Grimes - BCS	4	Westerberg - Pitt	11
IBM	1	Whelan - Philips	1
Imperial College	5	Will - Georgia Tech.	5
Johansson - Lund	2	Willoughby - IBM	2
Jones - Harwell	6	Young - BCS	4
Loden - Lockheed	4	Zenios - Princeton	1

We also had a small collection of matrix generation programs and portable random number generators for supplying values when only the pattern was specified in the matrix file. We did not have a large demand for these and, in addition, the programs were not all of sufficient quality for wide distribution. It is likely that some will be included in future releases, but again we welcome your comment.

We already have acquired some additional problems which are not yet included in the set. These include some unsymmetric eigenvalue systems, large unsymmetric problems, very large symmetric problems, and some large least squares problems. We are currently organizing and documenting these prior to their inclusion in a subsequent release. In the meantime, if you feel that you have some interesting problems that you are happy to provide openly to the research community, please contact one of the authors of this document, preferably by email (to duff@cerfacs.fr, rgrimes@espresso.boeing.com, or jglewis@atc.boeing.com). We are also interested in your experience with the current collection and would like to hear the results of research with these matrices. Information such as the behaviour of a particular ordering algorithm on a subset of matrices in the collection is the kind of statistic we would like to include in our matrix documentation. Relative performance statistics are also useful. However, we do not wish specific information on the performance of program A on machine B with matrix C.

Table 3: Discipline for Matrices

acoustic scattering	4	demography	3	network flow	1
air traffic control	1	economics	11	numerical analysis	4
astrophysics	2	electric power	18	oceanography	4
biochemical	2	electrical engineering	1	petroleum engineering	19
chemical eng.	16	finite elements	50	reactor modelling	3
chemical kinetics	14	fluid flow	6	statistics	1
circuit physics	1	laser optics	1	structural engineering	95
computer simulation	7	linear programming	16	survey data	11

Table 4: Keywords for Matrices

buckling	5	normal equations	2	power network	15
eigenvalue	50	ode	16	reservoir simulation	19
emitter circuit	1	optimal power flow	3	Scotland survey	1
England survey	1	optimization	1	siesmic studies	3
Facsimile	14	orderings	72	statics	26
Holland survey	2	original Harwell	36	stiff	15
iterative methods	12	overdetermined	5	stiffness	38
least squares	4	p4	3	stress analysis	1
mass	23	pde	1	Sudan survey	1
Navier-Stokes	6	plant model	11	U. K. survey	2

Acknowledgment

We are happy to acknowledge the help of Eric Simonnet of ENSEEIHT, Toulouse in preparing the TeX files for the matrix documentation while he was a stagiaire at CERFACS.

References

[1] Duff, I. S., Grimes, R. G., and Lewis, J. G. Sparse matrix test problems. *ACM Trans. Math. Softw.*, 15:1–14, 1989.

A Sparse Matrix Format

We use two compact formats in our collection, one for arbitrary matrices in standard sparse matrix formulation and another to represent unassembled finite-element matrices in an elemental formulation. Each matrix in the collection is held as a sequence of formatted records that can be read conveniently into Fortran arrays. In §A.1 to §A.3, we describe the resulting matrix representation. In §A.4, we describe the format itself and give two example programs that read it.

A.1 Standard sparse matrix format

The standard sparse matrix format is column-oriented. That is, the matrix is represented by a sequence of columns. Each column is held as a sparse vector, represented by a list of the row indices of the entries in an integer array and a list of the corresponding values in a separate numerical array. A single integer array and a single numerical array are used to store the row indices and the values, respectively, for all of the columns. (Throughout, we use the term "numerical" in a generic sense so that it should be read as a Fortran real, double precision, complex, or double precision complex as appropriate). Data for each column are stored in consecutive locations, the columns are stored in order, and there is no space between the columns. A separate integer array holds the location of the first entry of each column and the first free location. For symmetric and Hermitian matrices, we store only the entries of the lower triangle (including the diagonal). For skew symmetric matrices, we hold only the strict lower triangle.

We illustrate the storage scheme with the following example. The 5×5 matrix

$$\begin{pmatrix} 1. & -3. & 0 & -1. & 0 \\ 0 & 0 & -2. & 0 & 3. \\ 2. & 0 & 0 & 0 & 0 \\ 0 & 4. & 0 & -4. & 0 \\ 5. & 0 & -5. & 0 & 6. \end{pmatrix}$$

would be stored in the arrays COLPTR (location of first entry), ROWIND (row indices), and VALUES (numerical values) according to the following prescription:

Subscripts	1	2	3	4	5	6	7	8	9	10	11
COLPTR	1	4	6	8	10	12					
ROWIND	1	3	5	1	4	2	5	1	4	2	5
VALUES	1.	2.	5.	-3.	4.	-2.	-5.	-1.	-4.	3.	6.

We can generate column 5, say, by observing that its first entry is in position COLPTR(5) = 10 of arrays ROWIND and VALUES. This entry is in row ROWIND(10) = 2 and has value VALUES(10) = 3. Other entries in column 5 are found by scanning ROWIND and VALUES to position COLPTR(6)-1, that is position 11. Thus, the only other entry in column 5 is in row ROWIND(11) = 5 with value VALUES(11) = 6.

A.2 Finite-element matrices in unassembled format

Matrices arising in finite-element applications are usually assembled from numerous small elemental matrices. Our collection includes a few sparse matrices in original unassembled form. The storage of the unassembled matrices is analogous to the explicit form above, which stores each matrix as a list of matrix columns. The elemental representation stores the matrix as a list of elemental matrices. Each elemental matrix is represented by a list of the row/column indices (variables) associated with the element and by a small dense matrix giving the numerical values by columns (in the symmetric case only the lower triangular part). The lists of indices are held contiguously, just as for the lists of row indices in the standard format. The dense matrices are held contiguously in a separate array, with each matrix held by columns. Our representation does not hold the pointers to the beginning of the numerical values for each element, even though there is not a 1-1 correspondence between the arrays of integer and numerical values. These pointers can be created from the index start pointers (ELTPTR) after noting that an element with ν variables has ν^2 numerical values ($\nu \times (\nu+1)/2$ in the symmetric case).

We illustrate the elemental storage scheme with a small example. Consider a 5×5 symmetric matrix

$$\begin{pmatrix} 5. & 0. & 0. & 1. & 2. \\ 0. & 4. & 3. & 0. & 6. \\ 0. & 3. & 7. & 8. & 1. \\ 1. & 0. & 8. & 9. & 0. \\ 2. & 6. & 1. & 0. & 10. \end{pmatrix}$$

generated from four elemental matrices,

$$\begin{array}{c} \\ 1 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 4 \end{array} \\ \begin{pmatrix} 2. & 1. \\ 1. & 7. \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ 1 \\ 5 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 5 \end{array} \\ \begin{pmatrix} 3. & 2. \\ 2. & 8. \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ 2 \\ 3 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} 2 & 3 & 5 \end{array} \\ \begin{pmatrix} 4. & 3. & 6. \\ 3. & 5. & 1. \\ 6. & 1. & 2. \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} 3 & 4 \end{array} \\ \begin{pmatrix} 2. & 8. \\ 8. & 2. \end{pmatrix} \end{array}$$

where the variable indices are indicated by the integers before each row and above each column. This matrix would be stored in the arrays ELTPTR (location of first entry), VARIND (variable indices), and VALUES (numerical values) according to the following prescription:

Subscripts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
ELPTR	1	3	5	8	10										
VARIND	1	4	1	5	2	3	5	3	4						
VALUES	2.	1.	7.	3.	2.	8.	4.	3.	6.	5.	1.	2.	2.	8.	2.

A.3 Right-hand sides

Where the matrices originate in the solution of linear equations and the right-hand sides are available, the right-hand-side vectors are stored with the matrices. Usually the right-hand-side vectors are dense, in which case they are stored contiguously (as in ordinary

Fortran array storage). Multiple right-hand sides are stored as consecutive vectors, so that the right-hand sides are accessible as the columns of a Fortran array.

An alternative form is available in which right-hand sides are represented in the same format as the matrix. For unassembled matrices the associated right-hand sides can be represented by elemental contributions. Right-hand sides in elemental form are stored as a sequence of small dense matrices, each small matrix having as many columns as the number of right-hand sides, and with as many rows as the corresponding element in the matrix representation. Within each elemental right-hand side, the rows correspond to the entries in the variable index vector for that element.

The format for assembled sparse matrices is used to store sparse right-hand sides. Applications with sparse right-hand sides are less common, but the sparsity can be used to advantage in direct solution techniques. We only allow sparse right-hand sides for assembled matrices, in which case we store the right-hand sides exactly as a standard sparse matrix, with the same number of rows as the coefficient matrix and the same number of columns as right-hand sides.

We allow the specification of a starting guess for the solution of the problem and a vector that purports to be the exact solution. These can only be supplied as full arrays and only when right-hand side(s) are present. Either or both of these arrays can be present. The starting vector(s) precede the solution vector(s) if both are given and the number of such vectors must be equal to the number of right-hand sides.

A.4 Detailed formats

Our collection is held in an 80-column, fixed-length format for portability. Each matrix begins with a multiple line header block, which is followed by two, three, or four data blocks. The header block contains summary information on the storage formats and space requirements. From the header block alone, the user can determine how much space will be required to store the matrix. Information on the size of the representation in lines is given for ease in skipping past unwanted data.

If there are no right-hand-side vectors, the matrix has a four-line header block followed by two or three data blocks containing, in order, the column (or element) start pointers, the row (or variable) indices, and the numerical values. If right-hand sides are present, there is a fifth line in the header block and a fourth data block containing the right-hand side(s). The blocks containing the numerical values and right-hand side(s) are optional. The right-hand side(s) can be present only when the numerical values are present. If right-hand sides are present, then vectors for starting guesses and the solution can also be present; if so, they appear as separate full arrays in the right-hand side block following the right-hand side vector(s).

The first line contains the 72-character title and the 8-character identifier by which the matrix is referenced in our documentation. The second line contains the number of lines for each of the following data blocks as well as the total number of lines, excluding the header block. The third line contains a three character string denoting the matrix type as well as the number of rows, columns (or elements), entries, and, in the case of unassembled matrices, the total number of entries in elemental matrices. The fourth line contains the

variable Fortran formats for the following data blocks. The fifth line is present only if there are right-hand sides. It contains a one character string denoting the storage format for the right-hand sides as well as the number of right-hand sides, and the number of row index entries (for the assembled case). The exact format is given by the following, where the names of the Fortran variables in the subsequent programs are given in parenthesis:

Line 1 (A72,A8)

Col. 1 - 72 Title (TITLE)

Col. 73 - 80 Key (KEY)

Line 2 (5I14)

Col. 1 - 14 Total number of lines excluding header (TOTCRD)

Col. 15 - 28 Number of lines for pointers (PTRCRD)

Col. 29 - 42 Number of lines for row (or variable) indices (INDCRD)

Col. 43 - 56 Number of lines for numerical values (VALCRD)

Col. 57 - 70 Number of lines for right-hand sides (RHSCRD)
(including starting guesses and solution vectors if present)
(zero indicates no right-hand side data is present)

Line 3 (A3, 11X, 4I14)

Col. 1 - 3 Matrix type (see below) (MXTYPE)

Col. 15 - 28 Number of rows (or variables) (NROW)

Col. 29 - 42 Number of columns (or elements) (NCOL)

Col. 43 - 56 Number of row (or variable) indices (NNZERO)
(equal to number of entries for assembled matrices)

Col. 57 - 70 Number of elemental matrix entries (NELTVL)
(zero in the case of assembled matrices)

Line 4 (2A16, 2A20)

Col. 1 - 16 Format for pointers (PTRFMT)

Col. 17 - 32 Format for row (or variable) indices (INDFMT)

Col. 33 - 52 Format for numerical values of coefficient matrix (VALFMT)

Col. 53 - 72 Format for numerical values of right-hand sides (RHSFMT)

Line 5 (A3, 11X, 2I14) — *Only present if there are right-hand sides present*

Col. 1 Right-hand side type:
F for full storage or
M for same format as matrix

Col. 2 G if a starting vector(s) (Guess) is supplied. (RHSTYP)

Col. 3 X if an eXact solution vector(s) is supplied.

Col. 15 - 28 Number of right-hand sides (NRHS)

Col. 29 - 42 Number of row indices (NRHSIX)
(ignored in case of unassembled matrices)

The three character type field on line 3 describes the matrix type. The following table lists the permitted values for each of the three characters. As an example of the type field, RSA denotes that the matrix is real, symmetric, and assembled.

First Character:

R Real matrix

C Complex matrix

P Pattern only (no numerical values supplied)

Second Character:

S Symmetric

U Unsymmetric

H Hermitian

Z Skew symmetric

R Rectangular

Third Character:

A Assembled

E Elemental matrices (unassembled)

To formalize the logical block structure of the data, we have included two pieces of sample FORTRAN code for reading a matrix in the format of the sparse matrix test collection. Both codes assume the data comes from input unit LUNIT. Neither is a complete code. Real code should include error checking to ensure that the target arrays into which the data are read are large enough. The design allows the arrays to be read by a separate subroutine that can avoid the use of possibly inefficient implicit DO-loops. The first sample code is for the standard case, a sparse matrix in standard format with no right-hand sides.

```
C     ===================================================================
C     ... SAMPLE CODE FOR READING A SPARSE MATRIX IN STANDARD FORMAT
C     ===================================================================

      CHARACTER      TITLE*72 , KEY*8    , MXTYPE*3 ,
     1               PTRFMT*16, INDFMT*16, VALFMT*20, RHSFMT*20

      INTEGER        TOTCRD, PTRCRD, INDCRD, VALCRD, RHSCRD,
     1               NROW  , NCOL  , NNZERO, NELTVL

      INTEGER        COLPTR (*), ROWIND (*)

      REAL           VALUES (*)

C     ------------------------
C     ... READ IN HEADER BLOCK
C     ------------------------

      READ ( LUNIT, 1000 ) TITLE , KEY   ,
     1                     TOTCRD, PTRCRD, INDCRD, VALCRD, RHSCRD,
     2                     MXTYPE, NROW  , NCOL  , NNZERO, NELTVL,
     3                     PTRFMT, INDFMT, VALFMT, RHSFMT
 1000 FORMAT ( A72, A8 / 5I14 / A3, 11X, 4I14 / 2A16, 2A20 )

C     -------------------------
C     ... READ MATRIX STRUCTURE
C     -------------------------

      READ ( LUNIT, PTRFMT ) ( COLPTR (I), I = 1, NCOL+1 )

      READ ( LUNIT, INDFMT ) ( ROWIND (I), I = 1, NNZERO )

      IF  ( VALCRD .GT. 0 )  THEN

C         ----------------------
C         ... READ MATRIX VALUES
C         ----------------------

          READ ( LUNIT, VALFMT ) ( VALUES (I), I = 1, NNZERO )

      ENDIF
```

The second sample code illustrates the full generality of the representation.

```
C     ================================================================
C     ... SAMPLE CODE FOR READING A GENERAL SPARSE MATRIX, POSSIBLY
C         WITH RIGHT-HAND SIDE VECTORS
C     ================================================================

      CHARACTER      TITLE*72 , KEY*8    , MXTYPE*3 , RHSTYP*3,
     1               PTRFMT*16, INDFMT*16, VALFMT*20, RHSFMT*20

      INTEGER        TOTCRD, PTRCRD, INDCRD, VALCRD, RHSCRD,
     1               NROW  , NCOL  , NNZERO, NELTVL,
     2               NRHS  , NRHSIX, NRHSVL, NGUESS, NEXACT

      INTEGER        POINTR (*), ROWIND (*), RHSPTR (*), RHSIND(*)

      REAL           VALUES (*) , RHSVAL (*), XEXACT (*), SGUESS (*)

C     ------------------------
C     ... READ IN HEADER BLOCK
C     ------------------------

      READ ( LUNIT, 1000 ) TITLE , KEY   ,
     1                     TOTCRD, PTRCRD, INDCRD, VALCRD, RHSCRD,
     2                     MXTYPE, NROW  , NCOL  , NNZERO, NELTVL,
     3                     PTRFMT, INDFMT, VALFMT, RHSFMT

      IF  ( RHSCRD .GT. 0 )
     1    READ ( LUNIT, 1001 ) RHSTYP, NRHS, NRHSIX

 1000 FORMAT ( A72, A8 / 5I14 / A3, 11X, 4I14 / 2A16, 2A20 )
 1001 FORMAT ( A3, 11X, 2I14 )

C     -------------------------
C     ... READ MATRIX STRUCTURE
C     -------------------------

      READ ( LUNIT, PTRFMT ) ( POINTR (I), I = 1, NCOL+1 )

      READ ( LUNIT, INDFMT ) ( ROWIND (I), I = 1, NNZERO )

      IF  ( VALCRD .GT. 0 )  THEN

C         ---------------------
C         ... READ MATRIX VALUES
C         ---------------------
```

```
          IF  ( MXTYPE (3:3) .EQ. 'A' )  THEN
              READ ( LUNIT, VALFMT ) ( VALUES (I), I = 1, NNZERO )
          ELSE
              READ ( LUNIT, VALFMT ) ( VALUES (I), I = 1, NELTVL )
          ENDIF

C         -------------------------
C         ... READ RIGHT-HAND SIDES
C         -------------------------

          IF  ( NRHS .GT. 0 )  THEN

              IF  ( RHSTYP(1:1) .EQ. 'F' ) THEN

C                 ------------------------------
C                 ... READ DENSE RIGHT-HAND SIDES
C                 ------------------------------

                  NRHSVL = NROW * NRHS
                  READ ( LUNIT, RHSFMT ) ( RHSVAL (I), I = 1, NRHSVL )

              ELSE

C                 ---------------------------------------------
C                 ... READ SPARSE OR ELEMENTAL RIGHT-HAND SIDES
C                 ---------------------------------------------


                  IF (MXTYPE(3:3) .EQ. 'A') THEN

C                    ------------------------------------------------
C                    ... SPARSE RIGHT-HAND SIDES - READ POINTER ARRAY
C                    ------------------------------------------------

                     READ (LUNIT, PTRFMT) ( RHSPTR (I), I = 1, NRHS+1 )

C                    ---------------------------------------
C                    ... READ SPARSITY PATTERN FOR RIGHT-HAND
C                        SIDES
C                    ---------------------------------------

                     READ (LUNIT, INDFMT) ( RHSIND (I), I = 1, NRHSIX )

C                    -------------------------------------
C                    ... READ SPARSE RIGHT-HAND SIDE VALUES
C                    -------------------------------------

                     READ (LUNIT, RHSFMT) ( RHSVAL (I), I = 1, NRHSIX )
```

```
                ELSE

C                  ----------------------------------
C                  ... READ ELEMENTAL RIGHT-HAND SIDES
C                  ----------------------------------

                   NRHSVL = NNZERO * NRHS
                   READ (LUNIT, RHSFMT) ( RHSVAL (I), I = 1, NRHSVL )

                ENDIF

             END IF

             IF  ( RHSTYP(2:2) .EQ. 'G' ) THEN

C                ------------------------
C                ... READ STARTING GUESSES
C                ------------------------

                NGUESS = NROW * NRHS
                READ (LUNIT, RHSFMT) ( SGUESS (I), I = 1, NGUESS )

             END IF

             IF  ( RHSTYP(3:3) .EQ. 'X' ) THEN

C                ------------------------
C                ... READ SOLUTION VECTORS
C                ------------------------

                NEXACT = NROW * NRHS
                READ (LUNIT, RHSFMT) ( XEXACT (I), I = 1, NEXACT )
             END IF
          END IF
       END IF
```

The code above outlines the structure of the data. The interpretation of the row (or variable) index arrays will require knowledge of the matrix and right-hand side types, as read in this code.

B Matrices in the Collection

TITLE: ACOUST Acoustic Scattering

DESCRIPTION:

Source: David P. Young, Boeing Computer Services, Seattle, Washington.

Discipline: Dynamic analyses in structural engineering

Remarks: These are 4 complex symmetric matrices that arise in modelling the acoustic scattering phenomenon. The fundamental operator is the Helmholtz equation with varying k and Dirichlet boundary conditions. All four problems come from a grid with 29 by 29 interior points. Some interior points are held fixed as conductors. Five and nine-point difference formulae are used depending on the value of k at points in grid.

Accession: Spring 1984

MATRIX CHARACTERISTICS:

Type: Complex symmetric

Statistics:

Identifier	Order	Number of entries
YOUNG1C	841	4089
YOUNG2C	841	4089
YOUNG3C	841	3988
YOUNG4C	841	4089

PERFORMANCE STATISTICS:

Standard iterative methods, even with well chosen preconditioners, perform poorly on these problems.

REFERENCES:

None currently available (please submit some data)

TITLE: AIRTFC Air-traffic Control Model

DESCRIPTION:

Source: Stavros Zenios, Princeton University

Discipline: Air-traffic control.

Remarks: Symmetric matrix from the Air-Traffic Control Model. This matrix is the Hessian of the objective function. Several rows will be zero corresponding to variables not in the basis at the time the matrix was generated. Zenios solved a projection of the Hessian using conjugate gradient algorithms.

Accession: Autumn 1985

MATRIX CHARACTERISTICS:

Type: Symmetric matrix. Some zero rows and columns.

Statistics:

Identifier	Order	Number of entries
ZENIOS	2873	15032

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

Zenios, S.A. and Mulvey, J.M. (1986). Relaxation techniques for strictly convex network problems. Annals of Operations Research.

TITLE: ASTROPH Nonlinear astrophysics problems

DESCRIPTION:

Source:	Mats Carlsson, Institute of Theoretical Astrophysics, University of Oslo, Norway.
Discipline:	Radiative transfer and statistical equilibrium in astrophysics.
Remarks:	Was using a frontal code to solve equations. Cray Research (UK) Ltd suggested trying the Harwell code MA32.
Accession:	March 1985.

MATRIX CHARACTERISTICS:

Type: Unsymmetric but with variable band characteristics.

Statistics:

Identifier	Order	Number of entries
MCCA	180	2659
MCFE	765	24382

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

Scharmer, G.B. and Carlsson, M. (1985). A new approach to multi-level non-LTE radiative transfer problems. *J.Comp.Phys.* **38**

Carlsson, M. (1985). Uppsala Observatory Report 33.

TITLE: BCSPWR Power network patterns

DESCRIPTION:

Source: Collected by B. Dembart and J. Lewis, Boeing Computer Services, Seattle, WA, USA in 1981.

Discipline: These ten patterns come from the sparse matrix representation of various power networks.

Remarks: Widely used as examples of network rather than grid problems.

Accession: Summer 1981.

MATRIX CHARACTERISTICS:

Type: Symmetric but not suitable for variable-band algorithms.

Statistics: Set of ten square matrices of varying orders and density. BCSPWR06, 07, 08 and 09 represent different models of the same network; they each provide a viewpoint from some subregion within the network, with greater detail nearer that viewpoint.

Identifier	Description	Order	Number of entries
BCSPWR01	Standard IEEE Test System - New England	39	85
BCSPWR02	Small Test System - WSCC	49	108
BCSPWR03	IEEE Standard 118 Bus Test Case	118	297
BCSPWR04	Equivalenced Representation of US Network	274	943
BCSPWR05	Equiv. Representation of Western US	443	1033
BCSPWR06	Western US Power Network - 1454 Bus	1454	3377
BCSPWR07	Western US Power Network - 1612 Bus	1612	3718
BCSPWR08	Western US Power Network - 1624 Bus	1624	3837
BCSPWR09	Western US Power Network - 1723 Bus	1723	4117
BCSPWR10	Eastern US Power Network - 5300 Bus	5300	13571

PERFORMANCE STATISTICS:

see references

REFERENCES:

Lewis, J.G. and Poole, W.G. (1980). Ordering algorithms applied to sparse matrices in electrical power problems. In Erisman, Neves, and Dwarakanath (1980), 115-124.

Lewis, J.G. and Simon, H.D. (1986). The impact of hardware gather/scatter on sparse Gaussian elimination. Report ETA TR-33, ETA Division, Boeing Computer Services, Seattle, Washington.

Liu, J.W.H. (1985). On the storage requirement in the out-of-core multifrontal method for sparse factorization. Report CS-85-02, Department of Computer Science, York University, Ontario, Canada.

Liu, J.W.H. (1985). An adaptive out-of-core Cholesky factorization scheme. Report CS-85-05 Department of Computer Science, York University, Ontario, Canada.

Smyth, W.F. and Dunn, J. (1986). Results of tests on matrix bandwidth and profile reduction algorithms. Report, Department of Computer Science and Systems, McMaster University, Ontario, Canada.

TITLE: BCSSTRUC1 BCS Structural Engineering Matrices (eigenvalue problems)

DESCRIPTION:

Source: John Lewis, Boeing Computer Services, Seattle, Washington.

Discipline: Dynamic analyses in structural engineering

Remarks: These matrices all represent dynamic analyses in structural engineering. They have been extracted from various structural engineering packages such as GT-STRUDL, MSC/NASTRAN, and BCS ATLAS. All of these matrices come in pairs. The first matrix, K, is the stiffness matrix while the second, M, is the mass matrix for the dynamic modelling of structures. Structural engineering requires the computations of a few modes, usually the lowest, of the generalized eigenvalue problem, $Kx = \lambda Mx$. Most of the matrices were extracted in the years 1980 to 1982.

Some of the collected problems demonstrate the effect of standard structural engineering techniques. BCSSTK02 and BCSSTM02 are the result of applying "static condensation" to the oil rig model represented by BCSSTK04 and BCSSTM04. Static condensation can be applied in cases where the mass matrix is singular to reduce the problem order while preserving the spectrum. However, the reduced stiffness matrix is usually dense, which is the case here. Good sparse eigenvalue codes should be able to solve a large sparse problem much more quickly than a dense code can solve the reduced problem of order one-half or one-third the original. This problem is probably too small to demonstrate that effect.

Matrices BCSSTK06 and BCSSTM06 represent the "lumped" (diagonal) mass formulation for the same problem for which BCSSTK07 and BCSSTM07 form the "consistent" mass formulation. BCSSTK11, BCSSTM11, BCSSTK12, and BCSSTM12 represent the lumped and consistent mass formulation for an ore car model. In both cases, the consistent mass formulations lead to non-diagonal mass matrices. The eigenvalues from the two formulations of a model should be similar, but not necessarily equal.

Matrices BCSSTK08 and BCSSTM08 have several clusters of eigenvalues where a doubleton (eigenvalue with multiplicity 2) is very close to a third eigenvalue.

Accession: Summer 1982.

MATRIX CHARACTERISTICS:

Type: Symmetric, M matrices are positive semi-definite.

Statistics:

Identifier	Description		Order	Number of entries
BCSSTK01	Small Test Problem	-K	48	224
BCSSTM01		-M	48	48
BCSSTK02	Oil Rig - Statically Condensed	-K	66	2211
BCSSTM02		-M	66	66
BCSSTK03	Small Test Structure	-K	112	376
BCSSTM03		-M	112	112
BCSSTK04	Oil Rig - Not Condensed	-K	132	1890
BCSSTM04		-M	132	132
BCSSTK05	Transmission Tower	-K	153	1288
BCSSTM05		-M	153	153
BCSSTK06	Medium Test Problem - Lumped Mass	-K	420	4140
BCSSTM06		-M	420	420
BCSSTK07	Medium Test Problem - Consistent Mass	-K	420	4140
BCSSTM07		-M	420	3836
BCSSTK08	TV Studio	-K	1074	7017
BCSSTM08		-M	1074	1074
BCSSTK09	Square Plate Clamped	-K	1083	9760
BCSSTM09		-M	1083	1083
BCSSTK10	Buckling of a Hot Washer	-K	1086	11578
BCSSTM10		-M	1086	11589
BCSSTK11	Ore Car - Lumped Masses	-K	1473	17857
BCSSTM11		-M	1473	1473
BCSSTK12	Ore Car - Consistent Masses	-K	1473	17857
BCSSTM12		-M	1473	10566
BCSSTK13	Fluid Flow Generalized Eigenvalues	-K	2003	42943
BCSSTM13		-M	2003	11973

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

Lewis, J.G. and Grimes, R.G. (1981). Practical Lanczos algorithms for solving structural engineering eigenvalue problems. *Sparse matrices and their uses,* I.S Duff (ed.), Academic Press, New York and London, 349-355

Lewis, J.G. and Simon, H.D. (1984). Numerical experience with the spectral transformation Lanczos method. Report MM-TR-16, ETA Division, Boeing Computer Services, Seattle, Washington.

Liu, J.W.H. (1985). On the storage requirement in the out-of-core multifrontal method for sparse factorization. Report CS-85-02, Department of Computer Science, York University, Ontario, Canada.

Smyth, W.F. and Dunn, J. (1986). Results of tests on matrix bandwidth and profile reduction algorithms. Report, Department of Computer Science and Systems, McMaster University, Ontario, Canada.

TITLE: BCSSTRUC2 BCS structural engineering matrices (linear equations)

DESCRIPTION:

Source: Prof Mac Will, Georgia Institute of Technology.

Discipline: Static analyses in structural engineering

Remarks: These matrices are each a statics (linear equation) problem arising from applications of the GT-STRUDL structural engineering code. They were collected as a set of benchmarks for out-of-core envelope factorization on limited memory computers (for example, CDC 6600 series).

Accession: Summer 1985

MATRIX CHARACTERISTICS:

Type: Symmetric positive definite

Statistics:

Identifier	Description	Order	Number of entries
BCSSTK14	Roof of Omni Coliseum, Atlanta	1806	32630
BCSSTK15	Module of an Offshore Platform	3948	60882
BCSSTK16	Corp. of Engineers Dam	4884	147631
BCSSTK17	Elevated Pressure Vessel	10974	219812
BCSSTK18	R.E.Ginna Nuclear Power Station	11948	80519

PERFORMANCE STATISTICS:

algorithms:

RCM = reverse Cuthill McKee (Sparspak)

AND = automatic nested dissection (Sparspak)

MMD = multiple minimum degree (Sparspak)

MA27 = minimum degree (Harwell MA27 code)

Nonzero entries in factor lower triangle				
Identifier	RCM	ND	MMD	MA27
BCSSTK15	1001906	594951	1707712	647274
BCSSTK16	622603	816998	812501	736294
BCSSTK17	3852413	1463496	1150523	994885
BCSSTK18	5896194	1648466	692872	650777

Nonzero entries required in-core for out-of-core factorization				
Identifier	MA27	MA27/Liu	MMD multifrontal	MMD general sparse
BCSSTK15	350695	345526	258540	188230
BCSSTK16	418895	195237	114462	105299
BCSSTK17	351886	306011	155504	91135
BCSSTK18	259600	189577	149058	190521

REFERENCES:

Lewis, J.G. and Simon, H.D. (1986). The impact of hardware gather/scatter on sparse Gaussian elimination. Report ETA TR-33, ETA Division, Boeing Computer Services, Seattle, Washington.

Liu, J.W.H. (1985). An adaptive out-of-core Cholesky factorization scheme. Report CS-85-05, Department of Computer Science, York University, Ontario, Canada.

TITLE: BCSSTRUC3 BCS structural engineering matrices (eigenvalue problems)

DESCRIPTION:

Source: John Lewis, Boeing Computer Services, Seattle, Washington.

Discipline: Dynamic analyses in structural engineering

Remarks: These generalized symmetric eigenproblems were extracted from various structural engineering packages such as GT-STRUDL, MSC/NASTRAN, and BCS ATLAS. They represent interesting problems encountered after the first set of structural engineering matrices was collected. Please refer to BCS Structural Engineering Matrices for more detail on these problems. These matrices were extracted in 1983 and 1984.

Problems numbered 19, 20 and 22 have very poorly conditioned stiffness matrices. Problem 21 is a standard textbook problem with multiple eigenvalues.

The smallest eigenvalue of BCSSTK25 has multiplicity 118.

Accession: Summer 1984 to spring 1985.

.

MATRIX CHARACTERISTICS:

Type: Symmetric generalized eigenproblem. M is a diagonal positive semi-definite matrix.

Statistics:

Identifier	Description		Order	Number of entries
BCSSTK19	Part of a Suspension Bridge	-K	817	3835
BCSSTM19		-M	817	817
BCSSTK20	Frame Within a Suspension Bridge	-K	485	1810
BCSSTM20		-M	485	485
BCSSTK21	Clamped Square Plate	-K	3600	15100
BCSSTM21		-M	3600	3600
BCSSTK22	Textile Loom Frame	-K	138	138
BCSSTM22		-M	138	138
BCSSTK23	Part of a 3D Globally Triang. Bldg.	-K	3134	24156
BCSSTM23		-M	3134	3134
BCSSTK24	Calgary Olympic Saddledome Arena	-K	3562	81736
BCSSTM24		-M	3562	3562
BCSSTK25	Columbia Center (Seattle), 76 storey	-K	15439	133840
BCSSTM25	skyscraper	-M	15439	15439

PERFORMANCE STATISTICS:

algorithms:

RCM = reverse Cuthill McKee (Sparspak)

AND = automatic nested dissection (Sparspak)

MMD = multiple minimum degree (Sparspak)

MA27 = minimum degree (Harwell MA27 code)

Nonzero entries in factor lower triangle				
Identifier	RCM	ND	MMD	MA27
BCSSTK24	534579	348739	294864	275360
BCSSTK25	2621675	2646153	1566229	1401129

Nonzero entries required in-core for out-of-core factorization				
Identifier	MA27	MA27/Liu	MMD multifrontal	MMD general sparse
BCSSTK24	85035	77367	68013	74900
BCSSTK25	853464	269237	216705	131832

REFERENCES:

Grimes, R.G., Lewis, J.G., and Simon, H.D. (1986). Experiences in solving large eigenvalue problems on the CRAY X-MP. Report ETA-TR-40, ETA Division, Boeing Computer Services, Seattle, Washington.

Liu, J.W.H. (1985). An adaptive out-of-core Cholesky factorization scheme. Report CS-85-05, Department of Computer Science, York University, Ontario, Canada.

Smyth, W.F. and Dunn, J. (1986). Results of tests on matrix bandwidth and profile reduction algorithms. Report, Department of Computer Science and Systems, McMaster University, Ontario, Canada.

TITLE: BCSSTRUC4 BCS structural engineering matrices (eigenvalue problems and linear equations)

DESCRIPTION:

Source: Andy Mera, Randy Cigel, and John Lewis, Boeing Computer Services, Seattle, Washington.

Discipline: Dynamic analyses in structural engineering

Remarks: These matrices have all been extracted from GT-STRUDL and MSC/NASTRAN The first two are generalized symmetric eigenproblems; the third is a linear equation problem. Please refer to BCS Structural Engineering Matrices for more detail on these problems. The first problem was collected in 1984; the latter two were collected in 1986.

BCSSTK26 and BCSSTM26 are from a vibration station. It is of interest because the number of eigenvalues needed for the seismic analysis was quite large – the lowest 197. The large number effectively defeats sparse eigenvalue routines that do not shift, and stands as a real benchmark for shifted algorithms.

Accession: Summer 1984 and spring 1986.

MATRIX CHARACTERISTICS:

Type: Symmetric generalized eigenproblem. Matrix **M** is positive definite. Positive definite linear equation.

Statistics:

Identifier	Description		Order	Number of entries
BCSSTK26	seismic analysis, nuclear power station	-K	1992	16129
BCSSTM26		-M	1992	1922
BCSSTK27	buckling analysis, symmetric half of	-K	1224	28675
BCSSTM27	an engine inlet from a modern Boeing jetliner	-M	1224	28675
BCSSTK28	solid element model, linear statics	-K	4410	111717

PERFORMANCE STATISTICS:

algorithms:

RCM = reverse Cuthill McKee (Sparspak)

AND = automatic nested dissection (Sparspak)

MMD = multiple minimum degree (Sparspak)

MA27 = minimum degree (Harwell MA27 code)

Nonzero entries in factor lower triangle				
Identifier	RCM	ND	MMD	MA27
BCSSTK28	967188	464925	342484	359932

Nonzero entries required in-core for out-of-core factorization				
Identifier	MA27	MA27/Liu	MMD multifrontal	MMD general sparse
BCSSTK28	101473	63519	58773	60467

REFERENCES:

None currently available (please submit some data)

TITLE: BCSSTRUC5 BCS structural engineering matrices (large eigenvalue problems)

DESCRIPTION:

Source: Boeing Computer Services.

Discipline: Structural engineering.

Remarks: These matrices were extracted from the MSC/NASTRAN or Boeing ATLAS structural engineering programs by Randy Cigel, Roger Grimes, John Lewis, and Ed Meyer. These are five very large problems encountered in detailed modelling of structures. Because of their size only the matrix patterns have been included in the collection. Given just cause and a willingness to deal with matrices with over 300,000 entries in the lower triangle the numerical values can be acquired from either Roger Grimes or John Lewis. These matrices were collected in 1986. The first problem is from a generalized symmetric eigenvalue problem; the pattern of the associated differential stiffness matrix is a subset of the pattern of the stiffness matrix. The next four are all linear equation problems.

Accession: Spring 1986.

MATRIX CHARACTERISTICS:

Type: Symmetric matrices, pattern only.

Statistics:

Identifier	Description	Order	Number of entries
BCSSTK29	Pattern for the stiffness matrix of a buckling model of a Boeing 767 rear pressure bulkhead	13992	316740
BCSSTK30	Pattern for the stiffness matrix from a statics model of an off-shore generator platform	28924	1036208
BCSSTK31	Pattern for the stiffness matrix from a statics model of an automobile component	35588	608502
BCSSTK32	Pattern for the stiffness matrix from a statics model of an automobile chassis	44609	1029655
BCSSTK33	Pattern for the stiffness matrix from a statics solid element model of a pin boss (part of an automobile steering mechanism)	8738	300321

PERFORMANCE STATISTICS:

algorithms:

RCM = reverse Cuthill McKee (Sparspak)

AND = automatic nested dissection (Sparspak)

MMD = multiple minimum degree (Sparspak)

MA27 = minimum degree (Harwell MA27 code)

Nonzero entries in factor lower triangle				
Identifier	RCM	ND	MMD	MA27
BCSSTK29	7374140	2548403	1680804	1849778
BCSSTK30	23242990	5627668	3814511	4571540
BCSSTK31	23641124	8873977	5272659	6014883
BCSSTK32	52170122	17244966	5201744	5855514
BCSSTK33	3799285	3179502	2538064	2583934

Nonzero entries required in-core for out-of-core factorization				
Identifier	MA27	MA27/Liu	MMD multifrontal	MMD general sparse
BCSSTK29	447876	334683	223251	301065
BCSSTK30	783941	442261	355918	410095
BCSSTK31	1447414	1447414	1021039	1278817
BCSSTK32	667385	483335	390897	658654
BCSSTK33	1103041	684727	879924	793601

REFERENCES:

Grimes, R.G., Lewis, J.G., and Simon, H.D. (1986). Experiences in solving large eigenvalue problems on the CRAY X-MP. Report ETA-TR-40, ETA Division, Boeing Computer Services, Seattle, Washington.

TITLE: BCSSTRUC6 BCS structural engineering matrices (linear equations)

DESCRIPTION:

Source: Ed Meyer, Boeing Computer Services.

Discipline: Structural Engineering.

Remarks: BLCKHOLE is an artificial finite-element model consisting of a finely gridded geodesic dome clamped to a coarsely gridded rectangular base. Because of the difference in gridding factors, the interface nodes on the base have very high order.
SSTMODEL is the elemental connectivity for the stiffness matrix of a 1960's design for a supersonic transport (the Boeing 2707).

Accession: Summer 1983.

MATRIX CHARACTERISTICS:

Type: Symmetric structures matrices. SSTMODEL is indefinite.

Statistics:

Identifier	Order	Number of entries (symmetric)
BLCKHOLE	2132	8502
SSTMODEL	3345	13047

PERFORMANCE STATISTICS:

For the BLCKHOLE example, the strange gridding plays havoc with orderings based on level structures, and is nearly a worst case for the Gibbs-Poole-Stockmeyer heuristic. Reverse Cuthill-Mckee and Gibbs-Poole-Stockmeyer are nearly a factor of two away from obtaining the minimum bandwith of this problem. General sparse methods fair well despite the strange gridding.

REFERENCES:

None currently available (please submit some data)

TITLE: CANNES Structures problems in aircraft design

DESCRIPTION:

Source: Lucien Marro, Programmation Scientifique, Aerospatiale Cannes, France.

Discipline: Finite-element structures problems in aircraft design.

Remarks: Sent to Iain Duff by Lucien Marro. He was using the test matrices as a testbed for his ordering codes being developed for his thesis.

Accession: June 3rd 1981.

MATRIX CHARACTERISTICS:

Type: Symmetric but not in ordering suitable for variable-band algorithms.

Statistics: Set of 18 square matrices of varying order and density. Their characteristics are as follows.

Identifier	Order	Number of entries
CAN 24	24	92
CAN 61	61	309
CAN 62	62	140
CAN 73	73	225
CAN 96	96	432
CAN 144	144	720
CAN 161	161	769
CAN 187	187	839
CAN 229	229	1033
CAN 256	256	1586
CAN 268	268	1675
CAN 292	292	1416
CAN 445	445	2127
CAN 634	634	3931
CAN 715	715	3690
CAN 838	838	5424
CAN 1054	1054	6625
CAN 1072	1072	6758

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

Marro, L. (1980). Méthodes de réduction de la largeur de bande et du profil efficace des matrices creuses. Thèse de 3ème cycle, Université de Nice.

Marro, L. (1986). A linear time implementation of profile reduction algorithms for sparse matrices. *SIAM J.Sci.Stat.Comput.* **7**, 1212-1231.

TITLE: CEGB Structural engineering matrices (unassembled)

DESCRIPTION:

Source: A.J. Donovan, CEGB, England.

Discipline: Finite-element calculations in structural engineering

Remarks: These matrices all represent static analyses in structural engineering. They have been extracted from the CEGB structural engineering package BERSAFE. All of these matrices are held as unassembled finite-element matrices.

Accession: Summer 1980.

MATRIX CHARACTERISTICS:

Type: Symmetric positive definite (unassembled).

Statistics:

Identifier	Description	Number of variables	Number of elements	Maximum element size
CEGB2919	Three dimensional cylinder with flange	2919	128	60
CEGB3024	Two-dimensional reactor core section	3024	551	16
CEGB3306	Framework problem essentially in two dimensions	3306	791	12
CEGB2802	Turbine blade	2802	108	60

PERFORMANCE STATISTICS:

See reference

REFERENCES:

Duff, I.S. and Reid, J.K. (1979). Performance evaluation of codes for sparse matrix problems. In *Performance evaluation of numerical software*, L.D Fosdick (ed.), North Holland, Amsterdam, New York, and London, 121-135.

TITLE: CHEMIMP Chemical engineering plant models

DESCRIPTION:

Source: David Bogle, Imperial College, London.

Discipline: Chemical engineering.

Remarks: Five matrices extracted from runs of the chemical engineering package SPEED UP. In each case, the matrix is the initial Jacobian approximation for a sparse nonlinear equation modelling a chemical process system.

Accession: August 1982.

MATRIX CHARACTERISTICS:

Type: Unsymmetric with many zeros on the diagonal. Some of the entries actually have the value zero. This happens when a procedure is used in which all outputs are incorrectly assumed to be a function of all inputs.

Statistics:

Identifier	Description	Order	Number of entries
IMPCOL A	Heat exchanger network	207	572
IMPCOL B	Cavett's process	59	312
IMPCOL C	Ethylene plant model	137	411
IMPCOL D	Nitric acid plant model	425	1339
IMPCOL E	Hydrocarbon separation problem	225	1308

PERFORMANCE STATISTICS:

These matrices can be significantly reduced by block triangularization methods.

REFERENCES:

None currently available (please submit some data)

TITLE: CHEMWEST Chemical Engineering Plant Models

DESCRIPTION:

Source: Art Westerberg, University of Pittsburgh.

Discipline: Chemical engineering.

Remarks: Eleven matrices extracted from modelling of chemical engineering plants.

Accession: Summer 1983

MATRIX CHARACTERISTICS:

Type: Unsymmetric with many zeros on the diagonal. Several of the explicitly held matrix entries also have the value zero.

Statistics:

Identifier	Description	Order	Number of entries
WEST0156	Simple Chemical Plant Model	156	371
WEST0167	Rigorous Model of a Chemical Stage	167	507
WEST0381	Multiply Fed Column, 24 Components	381	2157
WEST0132	Rigorous Flash Unit	132	414
WEST0067	Cavett Problem with 5 Components	67	294
WEST0655	16 Stage Column Section, Some Simplified	655	2854
WEST0479	8 Stage Column Section, All Rigorous	479	1910
WEST0497	Rigorous Flash Unit with Recycling	497	1727
WEST1505	11 Stage Column Section, All Rigorous	1505	5445
WEST2021	15 Stage Column Section, All Rigorous	2021	7353
WEST0989	7 Stage Column Section, All Rigorous	989	3537

PERFORMANCE STATISTICS:

Nine of these matrices can be significantly reduced by block triangularization methods.

REFERENCES:

Erisman, A.M., Grimes, R.G., Lewis, J.G., Poole, W.G. Jr., and Simon, H.D. (1983). Evaluation of orderings for unsymmetric sparse matrices. *SIAM J.Sci.Stat.Comput.* **8**, 600-624.

TITLE: CIRPHYS Circuit Physics modelling

DESCRIPTION:

Source:	J P Whelan, Circuit Physics and Applications Division, Philips Research Laboratories, Redhill, Surrey, England.
Discipline:	Computer random simulation of a circuit physics model
Remarks:	Was using MA28 and was concerned about the amount of fill-in. Example in test set is of order 991 but is indicative of structure of larger matrices (to order 3000) from the same modelling exercise.
Accession:	December 1978.

MATRIX CHARACTERISTICS:

Type: Unsymmetric but with variable band characteristics.

Statistics:

Identifier	Order	Number of entries
JPWH 991	991	6027

PERFORMANCE STATISTICS:

Fill-in using MA28 with u=0.1 is six times original number of nonzeros.

REFERENCES:

Erisman, A.M., Grimes, R.G., Lewis, J.G., Poole, W.G. Jr., and Simon, H.D. (1987). Evaluation of orderings for unsymmetric sparse matrices. *SIAM J.Sci.Stat.Comput.* **7**, 600-624.

TITLE: COUNTERX Small counter example matrices

DESCRIPTION:

Source: John Lewis, Boeing Computer Services, Seattle, Washington.

Discipline: Simple counter examples to Hellerman and Rarick algorithm

Remarks: These three matrix patterns were designed by Grimes and Lewis to demonstrate the type of breakdowns that can occur with the P^4 ordering. They also demonstrate how the P^5 ordering avoids the same type of breakdowns.

Other small problems that serve to demonstrate unusual behaviour of other algorithms are welcome.

Accession: Summer 1983.

MATRIX CHARACTERISTICS:

Type: Small unsymmetric patterns.

Statistics:

Identifier	Order	Number of entries
JGL009	9	50
JGL011	11	76
RGG010	10	76

PERFORMANCE STATISTICS:

The P^4 ordering reorders these matrices so that a zero is on the diagonal. JGL009 and JGL011 depend on fill-in during the factorization to provide a nonzero pivot when using Gaussian Elimination without pivoting. P^4 reorders RGG010 in such a way that a zero is placed on the diagonal and no fill occurs in that position leaving a "structural" zero for a pivot.

REFERENCES:

Erisman, A.M., Grimes, R.G., Lewis, J.G., and Poole, W.G. Jr. (1985). A structurally stable modification of Hellerman-Rarick's P^4 algorithm for reordering unsymmetric sparse matrices. *SIAM J.Numer.Anal.* **22**, 369-385.

TITLE: DWT Everstine's Collection

DESCRIPTION:

Source: Gordon Everstine, David W. Taylor Naval Ship Research and Development Center, Bethesda, MD, USA.

Discipline: Structural engineering

Remarks: This collection consists of thirty matrix patterns collected by Gordon Everstine of the David W. Taylor Naval Ship Research and Development Center, Bethesda, MD, USA. These patterns were collected from various US military and NASA users of NASA's structural engineering package NASTRAN for use as a benchmark collection for variable bandwidth reordering heuristics. They have been widely used in benchmarks; they are also of interest because 2- and 3-D plots are given in the reference below (Everstine 1979) for all thirty patterns.

Accession: Summer 1980.

MATRIX CHARACTERISTICS:

Type: Symmetric, patterns only.

Statistics:

Identifier	Order	Number of entries (symmetric)	Description
DWT 59	59	163	2D frame
DWT 66	66	193	Truss
DWT 72	72	147	Grillage
DWT 87	87	314	Tower
DWT 162	162	672	Plate with hole
DWT 193	193	1843	Knee prosthesis
DWT 198	198	795	Reinforced mast
DWT 209	209	976	Console
DWT 221	221	925	Hull-tank region
DWT 234	234	534	Tower with platform
DWT 245	245	853	Carriage
DWT 307	307	415	Power supply housing
DWT 310	310	1379	Hull with refinement
DWT 346	346	1786	Deckhouse
DWT 361	361	1657	Cylinder with cap
DWT 419	419	1991	Barge
DWT 492	492	1824	Piston shaft

Identifier	Order	Number of entries (symmetric)	Description
DWT 503	503	3265	Baseplate
DWT 512	512	2007	Submarine
DWT 592	592	2848	CVA bent
DWT 607	607	2869	Wankel rotor
DWT 758	758	3376	
DWT 869	869	4075	
DWT 878	878	4163	Plate with insert
DWT 918	918	4151	Beam with cutouts
DWT 992	992	8868	Mirror
DWT 1005	1005	4813	Baseplate
DWT 1007	1007	4791	
DWT 1242	1242	5834	Sea chest
DWT 2680	2680	13853	Destroyer

PERFORMANCE STATISTICS:

The first two references provide statistics on several variable-band and wavefront heuristics. The third reference gives the performance of several minimum degree reorderings on the largest of these problems.

REFERENCES:

Armstrong, B.A. (1986). Near minimal matrix profiles and wavefronts for testing nodal resequencing algorithms. *Int.J.Numer.Meth.Engng.* **21**, 1785-1790.

Everstine, G.C. (1979). A comparison of three resequencing lgorithms for the reduction of matrix profile and wavefront. *Int.J.Numer.Meth.Engng.* **14** , 837-853.

Lewis, J.G. (1982). Implementation of the Gibbs-Poole-Stockmeyer and Gibbs-King algorithms. *ACM Trans.Math.Softw.* **8**, 180-189 and 190-194.

Liu, J.W.H. (1985). Modification of the minimum degree algorithm by multiple elimination. *ACM Trans.Math.Softw.* **11**, 141-153.

Liu, J.W.H. (1985). On the storage requirement in the out-of-core multifrontal method for sparse factorization. Report CS-85-02, Department of Computer Science, York University, Ontario, Canada.

Marro, L. (1986). A linear time implementation of profile reduction algorithms for sparse matrices. *SIAM J.Sci.Stat.Comput.* **7**, 1212-1231.

Smyth, W.F. and Dunn, J. (1986). Results of tests on matrix bandwidth and profile reduction algorithms. Report, Department of Computer Science and Systems, McMaster University, Ontario, Canada.

TITLE: ECONAUS Australian Economic Models

DESCRIPTION:

Source: Ken Pearson, Department of Mathematics, La Trobe University, Melbourne, Australia.

Discipline: Economic modelling.

Remarks: These matrices come from problems solved using the economic modelling package GEMPACK. The principal linear equation solver used by this package is MA28.

Accession: October 24th, 1984.

MATRIX CHARACTERISTICS:

Type: Unsymmetric.

Statistics:

Identifier	Description	Order	Entries	Right-hand sides Number	Right-hand sides Entries
ORANI678	Economic model of Australia, 1968/69 data	2529	90158	116	297
MAHINDAS	Economic model of Victoria, Australia, 1880 data	1258	7682	55	162

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

Dixon, P.B., Parmenter, B.R., Sutton, J., and Vincent, D.P. (1982). ORANI: A Multisectoral Model of the Australian Economy. North Holland, Amsterdam.

Pearson, K.R. and Rimmer, R.J. (1984). An efficient method for the solution of large computable general equilibrium models. Sixth Biennnial Conference of the Simulation Society of Australia, August 1984. (To appear in Journal of the International Association for Mathematics and Computers in Simulation)

Pearson, K.R. and Rimmer, R.J. (1984). Sparse matrix methods on the VAX 11/780. Proceedings of the 24th European DECUS Symposium, Amsterdam, September 1984, 545-552.

Siriwaranda, A.M. (1985). A multisectoral general equilibrium model of tariff protection in the Colony of Victoria in 1880. Ph.D. Thesis, La Trobe University, Australia.

TITLE: ECONIEA Economic Models

DESCRIPTION:

Source: Daniel Szyld, Institute for Economic Analysis, New York University.

Discipline: Economic modelling

Remarks: The first three matrices (WM1, WM2, WM3) represent economic models of three of the fifteen regions used in the Input-Output model of the world economy by the Institute for Economic Analysis. The matrices are rectangular. Different square matrices are obtained by choosing the appropriate number of columns. Different choices will lead to different non-zero structures. An unfortunate choice may give a singular matrix but that will not be the case in general.

The next 6 matrices represent modelling of the economic transactions in the United States in 1972. MBEAUSE, MBEAFLW, and MBEACXC are the leading 496 by 496 principle minor of the matrices BEAUSE, BEAFLW, and BEACXC, respectively.
The rows of BEAUSE represent commodities and the columns represent industries.
The rows of BEAFLW represent industries and the columns represent industries.
The rows of BEACXC represent commodities and the columns represent commodities.

Accession: Summer 1982

MATRIX CHARACTERISTICS:

Type: Rectangular

Statistics:

Identifier	No. of rows	No. of columns	No. of entries
WM1	207	277	2909
WM2	207	260	2942
WM3	207	260	2948
BEAUSE	497	507	44551
BEAFLW	497	507	53403
BEACXC	497	506	50409
MBEAUSE	496	496	41603
MBEAFLW	496	496	49920
MBEACXC	496	496	49920

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

Szyld, D.B. (1981). Using sparse matrix techniques to solve a model of the world economy. *Sparse matrices and their uses,* I.S Duff (ed.), Academic Press, New York and London, 357-365.

Definitions and Conventions of the 1972 Input-Output Study. Staff Paper BEA-SP-80-034, Bureau of Economic Analysis, U.S. Dept. of Commerce, July 1980.

TITLE: FACSIMILE Chemical kinetics problems

DESCRIPTION:

Source: Alan Curtis, Computer Science and Systems Division, AERE Harwell.

Discipline: Chemical kinetics.

Remarks: Representative of the type of matrices which occur in spatially homogeneous problems from straight chemical kinetics calculations and mixed kinetics diffusion problems. Matrices generated by FACSIMILE stiff ode solver.

Three sets of matrices. Within each set there are three or four matrices from different timesteps in the solution of stiff ordinary differential equations in chemical kinetics. All matrices in each set have the same structure but quite different numerical values.

Accession: August 1983

MATRIX CHARACTERISTICS:

Type: Unsymmetric with diagonal entries larger than any in their row or column. The pattern is very scattered with no semblance of a band or any block structure. This means that, for the diffusion problems, the dependent species at one point in space are not ordered consecutively.

Statistics:

Identifier	Description	Order	Entries	Matrices in set
PSMOG	Straight chemical kinetics problem from atmospheric pollution studies. This is an example at the large end of the range of spatially homogeneous problems from straight chemical kinetics calculations.	183	1069	FS 183 1 FS 183 3 FS 183 4 FS 183 6

Identifier	Description	Order	Entries	Matrices in set
RCHEM	Mixed kinetics diffusion problem from radiation chemistry. 17 chemical species and one space dimension with 40 mesh points. The diffusion terms are large compared with the kinetics terms. However, 440 columns are identically those of the unit matrix which possibly provides a good test for methods which cannot exploit such a feature properly (for example, methods which assume symmetry).	680	2646	FS 680 1 FS 680 2 FS 680 3
STRAT	Mixed kinetics diffusion problem from study of ionization in the stratosphere with 38 chemical species. The diffusion terms are small compared with the kinetics terms but large compared with 1 (particularly for the last matrix in the set).	760	5976	FS 760 1 FS 760 2 FS 760 3

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

Erisman, A.M., Grimes, R.G., Lewis, J.G., Poole, W.G. Jr., and Simon, H.D. (1987). Evaluation of orderings for unsymmetric sparse matrices. *SIAM J.Sci.Stat.Comput.* **7**, 600-624

TITLE: GEMAT Optimal power flow problems

DESCRIPTION:

Source: Rob Burchett, General Electric Company, Schenectady, New York.

Discipline: Power flow modelling

Remarks: These matrices are from optimal power flow modelling. The first matrix, GEMAT1, is the Jacobian matrix for an approximately 2400 bus system in the Western United States. The rows represent the equality and some nonlinear constraints of the power flow problem. The first block of columns represents the standard angle and voltage variables of the power flow problem. The last block of columns is very sparse and represents the transformer taps and capacitors.

The initial basis for this problem is given by matrix GEMAT11. The basis after 100 optimal power flow iterations is given by matrix GEMAT12.

Accession: Summer 1984.

MATRIX CHARACTERISTICS:

Type: Rectangular matrices.

Statistics:

Identifier	Number of rows	Number of columns	Number of entries
GEMAT1	4929	10595	47369
GEMAT11	4929	4929	33185
GEMAT12	4929	4929	33111

PERFORMANCE STATISTICS:

This problem was brought to our attention by Mike Saunders. MINOS was used to solve this problem and the original ordering (P^4) in MINOS did not perform well on this problem because the initial basis is almost symmetric.

REFERENCES:

Erisman, A.M., Grimes, R.G., Lewis, J.G., Poole, W.G. Jr., and Simon, H.D. (1987). Evaluation of orderings for unsymmetric sparse matrices. *SIAM J.Sci.Stat.Comput.* **7**, 600-624

TITLE: GRENOBLE Simulation of computer systems

DESCRIPTION:

Source: François Cachard, University of Grenoble, France.

Discipline: Simulation studies in computer systems.

Remarks: Used as a testbed for ordering codes being developed for the thesis of François Cachard. The matrices were produced from runs of the package QNAP written by CII-HB for simulation modelling of computer systems.

Accession: June 3rd 1981.

MATRIX CHARACTERISTICS:

Type: Unsymmetric, mostly with a variable-band flavour.

Statistics:

Identifier	Order	Number of entries
GRE 115	115	421
GRE 185	185	1005
GRE 216A	216	876
GRE 216B	216	876
GRE 343	343	1435
GRE 512	512	2192
GRE 1107	1107	5664

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

Cachard, F. (1981). Logiciel numerique associé à une modélisation de systèmes informatiques. Thèse, Université Scientifique et Médicale de Grenoble, et l'Institut National Polytechnique de Grenoble.

Erisman, A.M., Grimes, R.G., Lewis, J.G., Poole, W.G. Jr., and Simon, H.D. (1987). Evaluation of orderings for unsymmetric sparse matrices. *SIAM J.Sci.Stat.Comput.* **7**, 600-624

TITLE: JAGMESH Graded L-shape patterns

DESCRIPTION:

Source: Alan George, University of Waterloo, Canada.

Discipline: Finite-element model problem

Remarks: These 9 matrix patterns arise from the application of triangular finite-element discretization of the heat conduction problem on variously shaped regions. The meshes are refined to yield patterns with approximately 1000 nodes.

These patterns were used by J. George and J. Liu to compare various reordering methods in the development of SPARSPAK during the late 1970's.

Accession: January 1978.

MATRIX CHARACTERISTICS:

Type: Symmetric, patterns only.

Statistics:

Identifier	Description	Order	Number of entries
JAGMESH1	small hole square	936	3600
JAGMESH2	graded L	1009	3937
JAGMESH3	plain square	1089	4225
JAGMESH4	large hole square	1440	5472
JAGMESH5	+ shaped domain	1180	4465
JAGMESH6	H shaped domain	1377	5185
JAGMESH7	3 hole problem	1138	4294
JAGMESH8	6 hole problem	1141	4303
JAGMESH9	pinched hole problem	1349	5225

PERFORMANCE STATISTICS:

See references for performance of SPARSPAK on these problems.

REFERENCES:

George, A. and Liu, J.W.H. (1981). *Computer solution of large sparse positive-definite systems.* Prentice-Hall, Englewood Cliffs, New Jersey.

Lewis, J.G. (1983). Numerical experiments with SPARSPAK. *SIGNUM Newsletter*, Association for Computing Machinery, New York **18** (3), 12-22.

Liu, J.W.H. (1985). On the storage requirement in the out-of-core multifrontal method for sparse factorization. Report CS-85-02, Department of Computer Science, York University, Ontario, Canada.

Smyth, W.F. and Dunn, J. (1986). Results of tests on matrix bandwidth and profile reduction algorithms. Report, Department of Computer Science and Systems, McMaster University, Ontario, Canada.

TITLE: LANPRO Lanczos with Partial Reorthogonalization

DESCRIPTION:

Source: Horst Simon, Boeing Computer Services, Seattle, Washington.

Discipline: Linear equations in structural engineering

Remarks: These are the seven positive definite matrices used as numerical examples in reference Simon (1984) on the solution of $Ax = b$ using the Lanczos algorithm with partial reorthogonalization. The first five are from finite-element approximations to problems in structural engineering. The last two are derived from finite-difference approximations to elliptic partial differential equations.

Accession: Spring 1982.

MATRIX CHARACTERISTICS:

Type: Positive definite symmetric matrices.

Statistics:

Identifier	Description	Order	Number of entries
NOS1	Biharmonic operator on a beam with one end free and one end fixed. 80 elements with 3 DOF per node.	237	627
NOS2	Same as above with 240 elements	957	2547
NOS3	Biharmonic operator on a rectangular plate with one side fixed and the others free	960	8402
NOS4	Beam structure	100	347
NOS5	3 story building with attached tower with each beam modelled as in NOS1	468	2820
NOS6	Poisson's equation in an L-shaped region, mixed boundary conditions	675	1965
NOS7	Diffusion equation with varying diffusivity in a 3D unit cube with Dirichlet boundary conditions	729	2673

PERFORMANCE STATISTICS:

The condition number of these problems range from 3.5×10^3 for NOS3 to 1.8×10^9 for NOS7. LANPRO (Lanczos with partial reorthogonalization) was 6 times faster than LANFRO (Lanczos with full reorthogonalization) for well-conditioned problems. LANPRO was only 1.5 times faster than LANFRO for ill-conditioned problems. See reference for more details.

REFERENCES:

Simon, H.D. (1984). The Lanczos algorithm with partial reorthogonalization. *Math.Comp.* **42**, 115-142.

Smyth, W.F. and Dunn, J. (1986). Results of tests on matrix bandwidth and profile reduction algorithms. Report, Department of Computer Science and Systems, McMaster University, Ontario, Canada.

TITLE: LAPLACE Finite-difference Laplacians

DESCRIPTION:

Source: Roger Grimes, Boeing Computer Services, Seattle, Washington.

Discipline: Partial differential equations

Remarks: These two matrices was generated from a nine point discretization of the Laplacian on the unit square with Dirichlet boundary conditions. A matrix generation program that generates Laplacians for five and nine point stars on other grids on the unit square can be obtained from Roger Grimes.

Accession: Summer 1983.

MATRIX CHARACTERISTICS:

Type: Symmetric positive definite matrix.

Statistics:

Identifier	Order	Number of entries (symmetric)
GR 05 05	25	97
GR 30 30	900	4322

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

Smyth, W.F. and Dunn, J. (1986). Results of tests on matrix bandwidth and profile reduction algorithms. Report, Department of Computer Science and Systems, McMaster University, Ontario, Canada.

TITLE: LAPU Laplace finite-element matrices (unassembled)

DESCRIPTION:

Source: Iain Duff, Harwell Laboratory

Discipline: Model finite-element calculations

Remarks: These matrices represent unassembled matrix from the finite-element discretization of Laplace's equation in a square. Two different grids are used and the matrices can be used for debugging software using elemental input. Only the patterns are supplied.

Accession: Summer 1980.

MATRIX CHARACTERISTICS:

Type: Symmetric patterns (unassembled).

Statistics:

Identifier	Description	Number of variables	Number of elements	Maximum element size
LAP 25	5 by 5 grid	25	16	4
LAP 900	30 by 30 grid	900	841	4

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

None currently available (please submit some data)

TITLE: LNS Fluid flow modelling

DESCRIPTION:

Source: Ian Jones, Harwell Laboratory.

Discipline: Fluid flow modelling.

Remarks: Six matrices arising in the solution of linearized Navier-Stokes equations for compressible flow, using velocity-pressure formulation.

The first three matrices correspond to an ordering by variable type with velocity variables (three dimensions) preceding temperature, preceding pressure variables. Since the continuity equations that are used to define the pressure do not contain explicit reference to the pressure the diagonal block is zero.

The second three matrices are permutations of the first three. The matrices have been permuted so that all variables at the same grid point are grouped together. One effect of this is to reduce the variable-bandwidth of the system.

Accession: Summer 1980.

MATRIX CHARACTERISTICS:

Type: Unsymmetric matrices but with nearly symmetric pattern.

Statistics:

Identifier	Order	Number of entries
LNS 131	131	536
LNS 511	511	2796
LNS 3937	3937	25407
LNSP 131	131	536
LNSP 511	511	2796
LNSP3937	3937	25407

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

None currently available (please submit some data)

TITLE: LOCKHEED Structural engineering matrices (unassembled)

DESCRIPTION:

Source: W.A. Loden, Lockheed Palo Alto Research Laboratory

Discipline: Finite-element calculations in structural engineering

Remarks: These matrices all represent static analyses in structural engineering. All of these matrices are held as unassembled finite-element matrices.

Accession: Summer 1980.

MATRIX CHARACTERISTICS:

Type: Symmetric positive definite (unassembled).

Statistics:

Identifier	Description	Number of variables	Number of elements	Maximum element size
LOCK1074	"gyro" cradle assembly	1074	323	24
LOCK 700	"porch" ocean-mining	700	324	18
LOCK2232	launch umbilical tower	2232	944	12
LOCK3491	"cross-cone" vehicle structure	3491	684	24

PERFORMANCE STATISTICS:

See reference

REFERENCES:

Jensen, P. S. and Loden, W. A. (1980). Supplementary study on the sensitivity of optimized structures. Report LMSC-D777859. Lockheed Palo Alto Research Laboratory.

TITLE: LSHAPE Graded L-shape patterns

DESCRIPTION:

Source: Alan George, University of Waterloo, Canada.

Discipline: Finite-element model problem

Remarks: These 12 matrix patterns arise from the application of triangular finite-element discretizations of the heat conduction problem on an L-shaped region. The mesh is progressively refined to yield 12 related patterns.

These patterns were used by J. George and J. Liu to compare various reordering methods in the development of SPARSPAK during the late 1970's.

Accession: January 1978.

MATRIX CHARACTERISTICS:

Type: Symmetric, patterns only.

Statistics:

Identifier	Order	Number of entries
LSHP 265	265	1009
LSHP 406	406	1561
LSHP 577	577	2233
LSHP 778	778	3025
LSHP1009	1009	3937
LSHP1270	1270	4969
LSHP1561	1561	6121
LSHP1882	1882	7393
LSHP2233	2233	8785
LSHP2614	2614	10297
LSHP3025	3025	11929
LSHP3466	3466	13681

PERFORMANCE STATISTICS:

See references for performance of SPARSPAK on these problems.

REFERENCES:

George, A. and Liu, J.W.H. (1981). *Computer solution of large sparse positive-definite systems.* Prentice-Hall, Englewood Cliffs, New Jersey.

Lewis, J.G. (1983). Numerical experiments with SPARSPAK. *SIGNUM Newsletter*, Association for Computing Machinery, New York **18** (3), 12-22.

Liu, J.W.H. (1985). On the storage requirement in the out-of-core multifrontal method for sparse factorization. Report CS-85-02, Department of Computer Science, York University, Ontario, Canada.

Smyth, W.F. and Dunn, J. (1986). Results of tests on matrix bandwidth and profile reduction algorithms. Report, Department of Computer Science and Systems, McMaster University, Ontario, Canada.

TITLE: LSQ Least squares problems in surveying

DESCRIPTION:

Source: Michael Saunders, SOL, Stanford University.

Discipline: Surveying.

Remarks: Four matrices from the least-squares solution of problems in surveying that were used by Michael Saunders and Chris Paige in the testing of LSQR. The second and fourth matrix have the same pattern as the first and third respectively but are much more ill-conditioned with respect to conjugate gradients.

Accession: Summer 1979

MATRIX CHARACTERISTICS:

Type: Rectangular.

Statistics:

Identifier	Number of rows	Number of columns	Number of entries
WELL1033	1033	320	4732
ILLC1033	1033	320	4732
WELL1850	1850	712	8758
ILLC1850	1850	712	8758

In each case one full right-hand side is supplied.

PERFORMANCE STATISTICS:

These matrices were used to test iterative methods.

REFERENCES:

None currently available (please submit some data)

TITLE: MANTEUFFEL Structural engineering matrices

DESCRIPTION:

Source: T.A. Manteuffel, Sandia Laboratories, Livermore

Discipline: Finite-element calculations in structural engineering

Remarks: The matrix is a condensed version from a model of structural deformation in three dimensions of a cylinder with varying thickness and holes. This matrix is held as an unassembled finite-element matrix.

Accession: Summer 1980

MATRIX CHARACTERISTICS:

Type: Symmetric positive definite (unassembled).

Statistics:

Identifier	Description	Number of variables	Number of elements	Maximum element size
MAN5976	Deformation of cylinder in 3-D	5976	784	20

PERFORMANCE STATISTICS:

See reference

REFERENCES:

Manteuffel, T. A. (1980). An incomplete factorization technique for positive definite linear systems. *Math. Comp.* **34**, 473-497.

TITLE: NNCENG Flow network problem

DESCRIPTION:

Source: R P Hornby, R & D Technology Department, National Nuclear Corporation Limited, Risley, Cheshire, England.

Discipline: Flow in networks.

Remarks: Problem arises from solving a system of equations obtained by conserving energy at each node of a network and equating the net pressure drop around any closed flow loop to zero. The flow field is turbulent and buoyancy forces are important so that the pressure drop Δp across any branch b is

$$\Delta p = \frac{R_b}{\rho_b} \mid W_b \mid W_b - \rho_b g \Delta h$$

where W_b is the branch flow, ρ_b the fluid density, R_b the branch resistance and h the node height. Linear systems from a Newton-Raphson linearization of this problem.

Accession: January 1983.

MATRIX CHARACTERISTICS:

Type: Symmetric pattern but unsymmetric matrix. Bands of nonzeros far from the diagonal and in the last rows and columns.

Statistics:

Identifier	Order	Number of entries
HOR 131	434	4710

PERFORMANCE STATISTICS:

An interesting characteristic is that the solution of the linearized system by MA28 is remarkably insensitive to changes in the threshold parameter.

REFERENCES:

None currently available (please submit some data)

TITLE: NUCL Nuclear reactor models

DESCRIPTION:

Source: National Nuclear Corporation

Discipline: Nuclear reactor core modelling

Remarks: These three matrices are derived from models of an advanced gas cooled reactor core, from the National Nuclear Corp. (UK). Their particular interest is the significant difference in factorization time between the original matrices and their transposes.

Accession: Spring 1982.

MATRIX CHARACTERISTICS:

Type: Unsymmetric.

Statistics:

Identifier	Order	Number of entries
NNC 261	261	1500
NNC 666	666	4044
NNC1374	1374	8606

PERFORMANCE STATISTICS:

Performance statistics from a CRAY-1S, CFT 1.09, for solution of a single right-hand side (times in CPU seconds)

	MA18		MA28	
Order	A	A^T	A	A^T
261	1.3		1.3	.5
666	14.3	16.6	13.8	5.0

REFERENCES:

None currently available (please submit some data)

TITLE: OILGEN Oil reservoir simulation - generated problems

DESCRIPTION:

Source: Roger Grimes, Boeing Computer Services, Seattle, Washington.

Discipline: Oil reservoir simulation

Remarks: Oil reservoir simulation is strongly oriented towards discretization of reservoirs with full 3D grids. Full grids provide a regular structure that can be exploited for reduced overhead and vector processing speeds. The restriction of a full grid can cause a substantial increase in the number of grid cells when finer refinement is used in the neighbourhood of coning wells and faults. Coalescing unnecessary cells into a coarser mesh in the areas of the reservoir away from coning wells and faults greatly reduces the number of cells. This can reduce the number of unknowns by a factor of 2.

Grimes developed a program to generate matrices similar to those encountered in three dimensional oil reservoir simulation. The program takes as input the oil reservoir described by 3D rectangular subregions that can overlap. Each subregion has associated material parameters and gridding specified.

A full 3D grid is generated and the associated Jacobian is computed. The full 3D grid may subdivide some subregions more than was requested on input. A reduced problem is generated by coalescing the extra cells caused by this subdivision into the single cell specified by the subregions original gridding.

Three matrices were generated by this program for the sparse matrix collection. The first problem was a model of a oil reservoir imbedded in a water aquifer with three coning wells. The reservoir was subdivided into an 8x10x5 grid. The subregions around the three wells were subdivided into 8x8x5, 4x4x5, and 4x4x5 grids respectively. The aquifer was subdivided into a 8x7x5 grid. The resulting full grid was 21x21x5. The matrix ORSREG 1 was the computed Jacobian. The unnecessary cells in the reservoir and aquifer caused by the finer refinement around the wells were coalesced to form ORSIRR 1.

Matrices for a second problem were also generated. The second problem was identical to the first except that the aquifer was divided into a 8x7x1 grid. The full grid problem was identical. The matrix resulting from the coalescing resulted in matrix ORSIRR 2.

Accession: Spring 1984.

MATRIX CHARACTERISTICS:

Type: Unsymmetric.

Statistics:

Identifier	Order	Number of entries
ORSREG 1	2205	14133
ORSIRR 1	1030	6858
ORSIRR 2	886	5970

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

None currently available (please submit some data)

TITLE: PLATZ Platzman's oceanographic models

DESCRIPTION:

Source: John Lewis, Boeing Computer Services, Seattle, Washington.

Discipline: Oceanic modelling

Remarks: The first two matrices are well known as difficult sparse symmetric eigenproblems. The larger matrix is a finite-difference model for the shallow wave equations for the Atlantic and Indian Oceans. The smaller matrix corresponds roughly to the North Atlantic Ocean. The original matrix is derived as the (negative) square of a purely imaginary skew-symmetric matrix. Hence, the eigenvalues occur in pairs (except for an isolated singleton at zero). A list of all of the eigenvalues of the smaller problem are given in the third reference. The third and fourth matrices are the skew-symmetric matrices whose squares are the first two matrices.

Accession: Summer 1975.

MATRIX CHARACTERISTICS:

Type: Eigenvalue problem. Last two matrices are purely imaginary skew-symmetric matrices. The first two matrices are their negative squares.

Statistics:

Identifier	Description	Order	# of entries (symmetric)
PLAT1919	full three ocean model	1919	17159
PLAT 362	North Atlantic submodel	362	3074
PLSK1919	skew-symmetric full model	1919	4831
PLSKZ362	skew symmetric North Atlantic submodel	362	880

PERFORMANCE STATISTICS:

See references.

The first three references describe the difficulties encountered in computing the desired eigenvalues in the mid 1970's. The problem is difficult because the desired natural modes correspond to interior eigenvalues of the matrices. The original problem was to find all eigenvalues in (.0001, .024), corresponding to natural modes that could contribute to global tides. The eigenvalues in (.000025, .0001) were also of interest, but were impossible to compute at that time. Modern shift and invert eigenvalue algorithms have little difficulty with these problems if they are capable of handling multiple eigenvalues. The unusual distribution of eigenvalues has, however, played havoc with some eigenvalue codes.

REFERENCES:

Cline, A.K., Golub, G.H., and Platzman, G.W. (1976). Calculations of normal modes of oceans using a Lanczos method. *Sparse matrix computations*, J.R Bunch and D.J Rose (eds.). Academic Press, London and New-York, 409-426.

Grimes, R.G., Lewis, J.G., and Simon, H.D. (1986). Experiences in solving large eigenvalue problems on the CRAY X-MP, Report ETA-TR-40, ETA Division, Boeing Computer Services, Seattle, Washington.

Lewis, J.G. (1977). Algorithms for sparse matrix eigenvalue problems. Report CS-77-595, Department of Computer Science, Stanford University, Stanford, California.

Lewis, J.G. and Simon, H.D. (1984). Numerical experience with the spectral transformation Lanczos method. Report MM-TR-16, ETA Division, Boeing Computer Services, Seattle, Washington.

Platzman, G.W. (1975). Normal modes of the Atlantic and Indian oceans. *J.Phys.Oceanography* **5**, 201-221.

Smyth, W.F. and Dunn, J. (1986). Results of tests on matrix bandwidth and profile reduction algorithms. Report, Department of Computer Science and Systems, McMaster University, Ontario, Canada.

TITLE: PORES Reservoir modelling

DESCRIPTION:

Source: John Appleyard, Harwell Laboratory.

Discipline: Reservoir modelling.

Remarks: Three matrices extracted from the PORES package for reservoir simulation.

Accession: Summer 1980.

MATRIX CHARACTERISTICS:

Type: Unsymmetric matrices but with symmetric pattern

Statistics:

Identifier	Order	Number of entries
PORES 1	30	180
PORES 2	1224	9613
PORES 3	532	3474

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

None currently available (please submit some data)

TITLE: PSADMIT Power systems admittance matrices

DESCRIPTION:

Source: Dan Tylavsky, Arizona State University.

Discipline: Power system networks.

Remarks: Four symmetric matrices used in the modelling of power system networks.

Accession: Summer 1985

MATRIX CHARACTERISTICS:

Type: Symmetric positive definite.

Statistics:

Identifier	Order	Number of entries
662 BUS	662	1568
494 BUS	494	1080
685 BUS	685	1967
1138 BUS	1138	2596

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

None currently available (please submit some data)

TITLE: PSMIGR Inter-county migration

DESCRIPTION:

Source: Paul Slater, University of California at Santa Barbara.

Discipline: Demography.

Remarks: Three matrices obtained from records containing counts of persons by sex and age who migrated across counties in the USA between 1965 and 1970. The 1970 15% sample data were used to tabulate the data. The first matrix gives the migration flow between counties in 1965 and 1970, so that entry (i,j) represents the migration from county j in 1965 to county i in 1970. For the second matrix, intra-county flows have been omitted (so that the diagonal is zero) and the matrix is in doubly-standardized (or doubly-stochastic) form. The third matrix is for doubly-standardized non-null diagonal flows.

Accession: November 1983.

MATRIX CHARACTERISTICS:

Type: Unsymmetric matrices. Quite dense but with a overlying block diagonal structure.

Statistics:

Identifier	Order	Number of entries
PSMIGR 1	3140	543162
PSMIGR 2	3140	540022
PSMIGR 3	3140	543162

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

Slater, P.B. (1983). Migration regions of the United States: two county-level 1965-70 analyses. Report, Community and Organization Research Institute, University of California, Santa Barbara, California.

TITLE: SAYLOR Saylor's petroleum engineering/reservoir simulation matrices

DESCRIPTION:

Source: Richard Kendall, Don Peaceman, Herb Stone, and Bill Watts, Exxon.

Discipline: Oil reservoir modelling

Remarks: These matrices were provided to Paul Saylor to be used as test cases for Saylor's work with Richardson's iteration with dynamic parameters (see reference).

The first matrix, SAYLR1, is a small linear system of 238 unknowns resulting from a 2D reservoir simulation based on field data. It is a simple problem and is used as a baseline comparison in Saylor's work.

The other 2 matrices, SAYLR3 and SAYLR4, result from the 3D simulation of reservoirs such that the shale poses vertical barriers to fluid flow and creates an almost random heterogeneity in the coefficient matrix. There are, in addition, enormous local contrasts in the transmissibility coefficients of the differential equation. These properties characterize matrices that are difficult to solve.

(NOTE: The matrix SAYLR3 is almost identical to the matrix SHERMAN1 which is one of the oil reservoir simulation challenge matrices contributed to this collection by Andy Sherman of Nolan and Associates.)

Accession: Summer 1984.

MATRIX CHARACTERISTICS:

Type: Symmetric matrices from partial differential equations.

Statistics:

Identifier	Description	Order	Number of entries (symmetric)
SAYLR1	2D Reservoir - 14 by 17	238	1128
SAYLR3	3D Reservoir - 10 by 10 by 10	1000	3750
SAYLR4	3D Reservoir - 33 by 6 by 18	3564	22316

PERFORMANCE STATISTICS:

Saylor compared Richardson's iteration with dynamic parameter estimation and the strongly implicit procedure (SIP) on these problems. Richardson's iteration was more effective than SIP. In a private communication Saylor indicated that any preconditioned conjugate gradient style method would be more effective than SIP or Richardson's iteration. See reference for details.

REFERENCES:

Saylor, P.E. (1981). Richardson's iteration with dynamic parameters and the SIP incomplete factorization for the solution of linear systems of equations. *Society of Petroleum Engineers J.,* 691-708

TITLE: SHERMAN Oil reservoir simulation challenge matrices

DESCRIPTION:

Source: Andy Sherman, Nolan and Associates, Houston, TX.

Discipline: Oil reservoir modelling.

Remarks: In the summer of 1984, Andy Sherman of Nolan and Associates, Houston, TX, USA, issued a challenge to the petroleum industry and the numerical analysis community for the fastest solution to a set of 5 systems of linear equations extracted from oil reservoir modelling programs. These are those five matrices. Each matrix arises from a three dimensional simulation model on a NX x NY x NZ grid using a seven-point finite-difference approximation with NC equations and unknowns per grid block. The corresponding right-hand side vector is also supplied.

Accession: Autumn 1984.

MATRIX CHARACTERISTICS:

Type: Symmetric matrices from partial differential equations.

Statistics:

Identifier	Description	Order	Number of entries (symmetric)
SHERMAN1	Black oil simulation, shale barriers (NX = NY = NZ = 10, NC = 1)	1000	3750
SHERMAN2	Thermal simulation with steam injection (NX = NY = 6, NZ = 5, NC = 5)	1080	23094
SHERMAN3	IMPES simulation of a black oil model (NX = 35, NY = 11, NZ = 13, NC = 1)	5005	20033
SHERMAN4	IMPES simulation with flow barriers (NX = 16, NY = 23, NZ = 3, NC = 1)	1104	3786
SHERMAN5	Fully implicit black oil mode (NX = 16, NY = 23, NZ = 3, NC = 3)	3312	20793

PERFORMANCE STATISTICS:

See reference.

REFERENCES:

Simon, H.D. (1985). Incomplete **LU** preconditioned conjugate-gradient-like methods in reservoir simulation. Proceeding of the Eighth SPE Symposium on Reservoir Simulation, Dallas, Feb.10-13, 387-396.

TITLE: SMTAPE Original Harwell sparse matrix test collection

DESCRIPTION:

Source: Harwell Laboratory, England.

Discipline: Variety of disciplines .. see table below.

Remarks: A collection of sparse matrices was begun by Curtis and Reid at Harwell in the early 1970's and was later extended by Duff into the present collection. These 36 matrices and matrix patterns come from a wide range of disciplines. A major objective of the test collection has been to represent important features of practical problems. Sparse matrix characteristics (such as average density of entries per row, pattern of the entries, symmetry, and matrix size) can differ among matrices arising from, for example, structural analysis, circuit design, or linear programming. The test problems, though varying widely in their characteristics, have very distinctive patterns. For the symmetric matrices, we record only the entries on and below the diagonal.

Accession: Summer 1978.

MATRIX CHARACTERISTICS:

Type: Various structures, see table.

Statistics:

Identifier	Order	Number of entries	Description
LUNDA	147	1298	Finite-element stiffness and mass matrices of
LUNDB	147	1294	generalized eigenvalue problem (T. Johansson of Lunds Datacentral, Lund, Sweden)
ERIS1176	1176	9864	Pattern of large electrical network (A. M. Erisman, Boeing Computer Services, Seattle, USA)
GENT113	113	655	Pattern of a matrix arising from a statistical application (W. M. Gentleman, Waterloo, Canada)
IBM32	32	126	Pattern of matrix advertising 1971 IBM conference on sparse matrices
CURTIS54	54	291	Pattern of matrix from the solution of a stiff set of biochemical ordinary differential equations (A. R. Curtis, Harwell, England)
WILL57	57	281	Pattern of Jacobian matrix associated with an emitter-follower-current switch circuit (Willoughby 1971)
WILL199	199	701	Pattern of a stress-analysis matrix (Willoughby 1971)

Identifier	Order	Number of entries	Description
ASH292	292	1250	Patterns of normal matrices associated with least-squares adjustment of survey data (V.Ashkenazi, Nottingham University, England)
ASH85	85	304	
ARC130	130	1282	Jacobian matrix of a set of ordinary differential equations associated with a laser problem (A. R. Curtis, Harwell, England)
SHL 0	663	1687	Basis matrices obtained at various stages of the application of the simplex method to two linear programming problems (M. A. Saunders, Systems Optimization Laboratory, Stanford University, USA)
SHL 200	663	1726	
SHL 400	663	1712	
STR 0	363	2454	
STR 200	363	3068	
STR 400	363	3157	
STR 600	363	3279	
BP 0	822	3276	
BP 200	822	3802	
BP 400	822	4028	
BP 600	822	4172	
BP 800	822	4534	
BP 1000	822	4661	
BP 1200	822	4726	
BP 1400	822	4790	
BP 1600	822	4841	
ASH219	219 x 85	438	Surveys of United Kingdom and Holland (V. Ashkenazi, Nottingham University, England)
ASH958	958 x 292	1916	
ASH331	331 x 104	662	
ASH608	608 x 188	1216	
ABB313	313 x 176	1557	Survey of Sudan (M. Abbas, Newcastle University, England)
FS 541 1	541	4285	Four matrices having the same pattern but varying conditioning, which arose at different stages of FACSIMILE (a stiff ODE package) in solving an atmospheric pollution problem involving chemical kinetics and two-dimensional transport (A. R. Curtis, Harwell, England)
FS 541 2			
FS 541 3			
FS 541 4			

PERFORMANCE STATISTICS:

See references.

REFERENCES:

Duff, I.S. and Reid, J.K. (1974). A comparison of sparsity orderings for obtaining a pivotal sequence in Gaussian elimination. *J.Inst.Maths.Applics.* **14**, 281-291.

Duff, I.S. and Reid, J.K. (1979). Performance evaluation of codes for sparse matrix problems. In *Performance evaluation of numerical software*, L.D. Fosdick (ed.), North Holland, Amsterdam, New York, and London, 121-135.

Duff, I.S., Erisman, A.M., and Reid, J.K. (1986). *Direct Methods for Sparse Matrices.* Oxford University Press.

Erisman, A.M., Grimes, R.G., Lewis, J.G., Poole, W.G. Jr., and Simon, H.D. (1987). Evaluation of orderings for unsymmetric sparse matrices. *SIAM J.Sci.Stat.Comput.* **7**, 600-624.

Willoughby, R.A. (1971). Sparse matrix algorithms and their relation to problem classes and computer architecture. In *Large sparse sets of linear equations,* J.K. Reid (ed.), Academic Press, New York and London, 255-277.

TITLE: STEAM Enhanced oil recovery

DESCRIPTION:

Source: Roger Grimes, Boeing Computer Services, Seattle, Washington.

Discipline: Oil recovery

Remarks: These three matrices were extracted from a program simulating enhanced oil recovery using injected steam.

Matrix STEAM1 represents a finite-difference discretization of a 4 by 4 by 5 grid with 3 variables at each grid point.

Matrix STEAM2 represents a finite-difference discretization of a 5 by 5 by 6 grid with 4 variables at each grid point.

Matrix STEAM3 represents a finite-difference discretization of a one dimensional grid with 20 grid points and 4 variables at each grid point.

Accession: Spring 1983

MATRIX CHARACTERISTICS:

Type: Symmetric positive definite

Statistics:

Identifier	Order	Number of entries
STEAM1	240	3762
STEAM2	600	13760
STEAM3	80	928

PERFORMANCE STATISTICS:

None currently available (please submit some data)

REFERENCES:

None currently available (please submit some data)

TITLE: WATT Petroleum engineering

DESCRIPTION:

Source: John Somerville, Department of Petroleum Engineering, Heriot-Watt University, Edinburgh, Scotland.

Discipline: Petroleum Engineering

Remarks: The values of the coefficients can vary by as much as 10^{14} and the use of automatic scaling routines were giving singular matrices. Fill-in was also higher than expected.

Accession: December 1983

MATRIX CHARACTERISTICS:

Type: Unsymmetric but with nearly a narrow band five diagonal structure.

Statistics:

Both matrices are essentially two versions of the same problem.

Identifier	Order	Number of entries
WATT 1	1856	11360
WATT 2	1865	11550

PERFORMANCE STATISTICS:

See remarks

REFERENCES:

None currently available (please submit some data)

Index